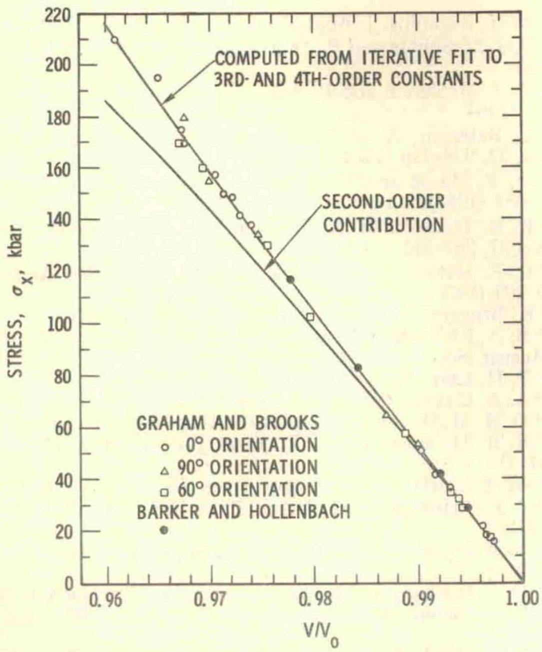

FIG. 1. The experimental shock-compression data for sapphire are plotted for comparison to the stress–volume relation computed from the third- and fourth-order elastic constants as determined in the text. The second-order constant contribution is taken from ultrasonic measurements of Gieske and Barsch.[35]

the room-temperature data but are included for completeness. The individual datum points of the various shock-compression experiments are compared to the stress–volume relation calculated from the high-order constants as computed from the data of Graham and Brooks in Fig. 1.

In analyzing the shock-compression data, it is interesting to note the compression ranges over which the various elastic constants made significant contributions to the stress amplitude. The second-order longitudinal constants give a good description to the observed stresses to compressions up to about 1%, whereas second- and third-order longitudinal constants give a good description to the observed stresses to compressions up to about 2%. The fourth-order contribution must be utilized to give a good description to the observed data from 2% to 4% compression. These observations emphasize the need for continuous data over a wide compression range if all the higher-order contributions are to be determined from shock-compression data.

B. Fused Quartz

Shock-compression studies of fused quartz have been accomplished from about 1 kbar to 65 kbar by Barker and Hollenbach[20] and up to 620 kbar by Wackerle.[37] At low pressures, the shock data show, in agreement with static compressibility measurements,[38] an increasing compressibility with pressure. This is contrary to the normal behavior of most solids and makes the Hugoniot elastic limit difficult to define. By employing both stress loading and stress unloading, Barker and Hollenbach found that fused quartz responds as an elastic solid to 38 kbar and, to a good approximation, the samples seemed elastic to 65 kbar.

Although the second-order constants of fused quartz have been measured by several investigators,[2,39] these constants are known to depend strongly on density.[7] Since they have not been measured on the fused quartz used in the shock-compression investigation, $\rho_0=2.201$ $g\cdot cm^{-3}$, the velocity of the leading edge of the stress pulse is used to determine the second-order constant. With the resulting value of $C_{11}=774.0$ kbar, the iterative procedure applied to the fused-quartz data to 38 kbar gives $C_{111}=+(5.5\pm0.1)\times10^3$ kbar and $C_{1111}=+(110\pm10)\times10^3$ kbar. The constants are compared to the ultrasonic data of Bogardus[2] in Table II.

The shock data above 38 kbar illustrate one particular problem with high-order constant measurements at large compressions. At stresses greater than 38 kbar, the bulk modulus begins to increase with pressure in the normal manner. This corresponds to the behavior observed by Bridgman[38] under static compression at 20 kbar since both the static and shock-compression results show that the change in bulk modulus begins to occur at a compression of 6%. Anderson and Dienes[40] have pointed out that a 6% volume compression would compress fused quartz to the volume of cristobalite, a well-known phase of SiO_2. Roy and Cohen[41] have observed a permanent densification and change in refractive index of fused silica for static pressures greater than 20 kbar. There is also evidence for a change in the strain dependence of the refractive index above 38 kbar from the shock data.[20] Thus, because there is evidence for a structural change at a volume compression of 6%, it appears that the high-order constant analysis should not be extended beyond 38 kbar.

The results indicate that the shock-compression technique yields a more accurate third-order constant than has heretofore been available. The fourth-order constant has been determined for the first time.

For the fused-quartz data, it is again instructive to consider the compression ranges over which the various-order elastic constants contribute significantly to the stress. The second-order constant gives a good fit up to

TABLE II. Comparison of high-order elastic constants of fused quartz.[a]

Method	Present work,[b] shock	Bogardus,[c] static ultrasonic
Maximum strain	6%	∼0.2%
C_{111}	5.5± 0.1	5.3±0.4
C_{1111}	110.0±10.0	···

[a] Units are 10^3 kbar. Temperatures are 25°C.
[b] Data from Ref. 20.
[c] Ref. 2.

compressions of 0.5%. Between 0.5% and 2.5%, the third-order constant gives a significant contribution, while the fourth-order constant gives a significant contribution for compressions from 2.5% to 6%.

IV. CONCLUSION

Analysis of the sapphire and fused-quartz shock-compression data demonstrate that the analytical method employed permits the determination of both third- and fourth-order longitudinal elastic constants. Although the third-order constants may be determined by other techniques, the fourth-order constants have been determined only by shock-compression techniques. The method is limited to solids that sustain large elastic compressions in uniaxial strain; however, a number of solids exhibit large Hugoniot elastic limits. The materials with known large Hugoniot elastic limits[25] include: sapphire, α quartz, MgO, Ge, Si, CdS, InSb, TiO_2, B_4C, BeO, and yttrium iron garnet. Thus, the method may be applied to a reasonably large number of solids of technical interest. Although shock-compression measurements have been performed on all these solids, the measurements are usually limited to several discrete stress–volume points, and these data are insufficient for the determination of third- and fourth-order constants

Even though there is some question as to the appropriateness of extending the theory to compression data to fourth order,[33] it is clear that the experimentally observed compressions of α quartz, sapphire, and fused quartz can be adequately described by the fourth-order constant development. Furthermore, it appears that shock-compression measurements can play a generally useful role in the determination of longitudinal third- and fourth-order elastic constants. If precise stress-versus-volume relations can be obtained under large elastic compressions, it appears that the finite-strain formulations can be given an evaluation. The present measurements are somewhat limited in accuracy, but it appears that the finite-strain formulation given here gives an appropriate description to both the ultrasonic and shock-compression data of sapphire and fused quartz.

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